

Introduction to Grammars

In the literary sense of the term, grammars denote syntactical rules for conversation in natural languages. Linguistics have attempted to define grammars since the inception of natural languages like English, Sanskrit, Mandarin, etc.

The theory of formal languages finds its applicability extensively in the fields of Computer Science. **Noam Chomsky** gave a mathematical model of grammar in 1956 which is effective for writing computer languages.

Grammar

A grammar **G** can be formally written as a 4-tuple (N, T, S, P) where –

- **N** or V_N is a set of variables or non-terminal symbols.
- **T** or Σ is a set of Terminal symbols.
- **S** is a special variable called the Start symbol, $S \in N$
- **P** is Production rules for Terminals and Non-terminals. A production rule has the form $\alpha \rightarrow \beta$, where α and β are strings on $V_N \cup \Sigma$ and least one symbol of α belongs to V_N .

Example

Grammar G1 –

$(\{S, A, B\}, \{a, b\}, S, \{S \rightarrow AB, A \rightarrow a, B \rightarrow b\})$

Here,

- **S, A,** and **B** are Non-terminal symbols;
- **a** and **b** are Terminal symbols
- **S** is the Start symbol, $S \in N$
- Productions, **P** : **S** \rightarrow **AB**, **A** \rightarrow **a**, **B** \rightarrow **b**

Example

Grammar G2 –

$((\{S, A\}, \{a, b\}, S, \{S \rightarrow aAb, aA \rightarrow aaAb, A \rightarrow \epsilon\})$

Here,

- **S** and **A** are Non-terminal symbols.
- **a** and **b** are Terminal symbols.
- ϵ is an empty string.
- **S** is the Start symbol, $S \in N$
- Production **P** : **S** \rightarrow **aAb**, **aA** \rightarrow **aaAb**, **A** \rightarrow ϵ

Derivations from a Grammar

Strings may be derived from other strings using the productions in a grammar. If a grammar **G** has a production $\alpha \rightarrow \beta$, we can say that **x α y** derives **x β y** in **G**. This derivation is written as –

$$x \alpha y \Rightarrow_G x \beta y$$

Example

Let us consider the grammar –

$G2 = (\{S, A\}, \{a, b\}, S, \{S \rightarrow aAb, aA \rightarrow aaAb, A \rightarrow \epsilon\})$

Some of the strings that can be derived are –

$S \Rightarrow \underline{a}Ab$ using production $S \rightarrow aAb$
 $\Rightarrow \underline{aa}Abb$ using production $aA \rightarrow aAb$
 $\Rightarrow \underline{aaa}Abbb$ using production $aA \rightarrow aAb$
 $\Rightarrow aaabbb$ using production $A \rightarrow \epsilon$